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Effect of Modal Dispersion on the Optimization of the Reflection Coefficient of Corrugated Planar Optical Waveguides

تأثير التشتت النمطى على معامل الأنعكاس الأمثل لموجهات الموجات الضوئية المتعرجة ذات المستوى الواحد

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الخلاصة :

فى هذا البحث تمت دراسة الخصائص العاكسة لموجهات الموجات الضوئية المتعرجة ذات المستوى الواحد التى تحقق شرط بــراج. و قــد استنبط شرط مميزى حديد لمعامل الأنعكاس الأمثل بطريقة تحليلية مع الأخذ فى الأعتبار التشتت النمطى لموجهات الموجات الضوئية. و قد وجد أن هذا الشرط لابد من تحقيقه قبل أستيفاء شرط براج. و الطريقة المستخدمة فى هذا البحث هى المصفوفة الأنتقالية التى تعتــبر أساس نظرية فلوكيه لموجهات الموجات الدورية.

Abstract:

The reflection characteristics of corrugated planar waveguides, satisfying Bragg condition, are investigated. A new eigenvalue criterion for optimum reflection coefficient is derived analytically taking into account the modal dispersion . This criterion must be met before the satisfaction of the well-known Bragg condition. A translation-matrix operator formalism (which is the core of Floquet's theory for periodic waveguides) is used in the analysis.

I. Introduction:

Over the past three decades and since the pioneering works of Kogelnik [1]. Yariv [2] . Wang [3] . Yariv [4] and Yeh [5] in the theory of distributed feedback waveguides and lasers. experimental and theoretical researches on guided wave Bragg gratings for laser diodes, quantum well lasers, optical amplification, soliton generation and propagation, optical bistability, dense wavelength division multiplexing (DWDM) and narrow-band high-reflectance filters are still continuing [6-15]. The periodic modulation of the refractive index of a planar optical waveguide (or fiber) acts like a selective mirror for the wavelengths that satisfy the Bragg condition. In other words, that periodic modulation forms a Bragg grating whose period and length, together with the strength of the modulation of the refractive index; determine the optical characteristics of the grating.

An example of periodic modulation of the refractive index is the planar corrugated waveguide shown in Fig. 1, where *the periodicity is limited to a cross section lying between the planes $x=d_1$ and $x=d_2$. The corrugations extend from $Z=0$ to $Z=L$.* That is, the corrugated section is a succession of short-length guides with varying thickness (variation between d_1 and d_2).

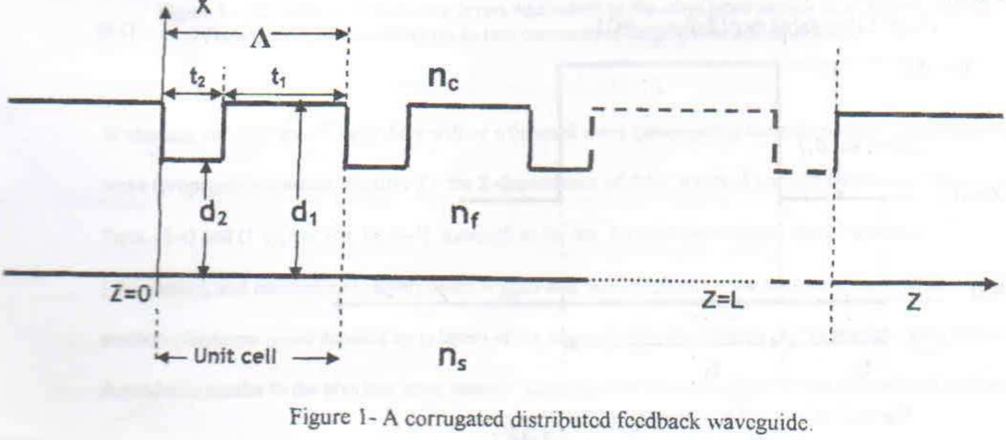


Figure 1- A corrugated distributed feedback waveguide.

This variation in thickness creates a periodic alternation (between $\beta_1$and β_2) of the propagation constant of the guided mode which propagates in that waveguide (we will consider the fundamental TE-mode i.e. its transverse electric field E_y is y-polarized and its transverse magnetic field H_x is x-polarized, and the Z-axis is the direction of propagation). The wave impedance (E_y/H_x) varies accordingly, as well as the field distribution in the transverse direction. The variation in wave impedance is the main factor that determines the reflection characteristics of the corrugated waveguide as pointed out by Yariv [4] , Wang [3] , and Hauss [16]. Of course the variation in the transverse field distribution causes some power loss [17] of the guided power via radiation in the superstrate , i.e. in $x>d_1$ and the substrate ($x<0$). However, by a rigorous mode matching technique (i.e. taking into account the effect of transverse mismatch in the mode structure), Marcuse [17] has shown that the reflection coefficient of a guided mode at a step discontinuity in a planar waveguide is dominated by the difference between the mode propagation constants.

In this paper, we used the translation-matrix operator formalism [4,5]; which is the foundation of Floquet's theory for wave propagation in periodic media, to show that the reflection coefficient of one period, i.e. a unit-cell, can be optimized when a certain eigenvalue criterion is satisfied and consequently the overall reflection coefficient of the corrugated section of the waveguide is optimized. This is achieved by a proper choice of the dispersion characteristics (before the satisfaction of the well known Bragg condition!) of the thick *and thin sections of the unit-cell shown in Fig. 2.*

II. Theoretical Analysis:

The problem of guided wave propagation in periodic structures can be handled by two main approaches: coupled-modes[1,2] and Floquet's theory [4,5] (which is based on the translation-matrix formalism) . Here we adopt the translation matrix formalism because, as will be shown later, it is more exact than the coupled-modes formalism and can handle a wide variety of periodic structures [4,5]. Fig. 2. shows a unit cell of the corrugated zone. TE and TM modes can be treated by the same method, which will be outlined hereafter, but for brevity we consider TE modes only. The electric fields of the fundamental TE modes in the thin and thick sections (which will be assumed single-mode) have the form [18]:

$$E_{y1}(x,z,t) = \mathcal{E}_1(x) \exp\{j(\beta_1 z - \omega t)\} \quad (1\text{-a})$$

$$E_{y2}(x,z,t) = \mathcal{E}_2(x) \exp\{j(\beta_2 z - \omega t)\} \quad (1\text{-b})$$

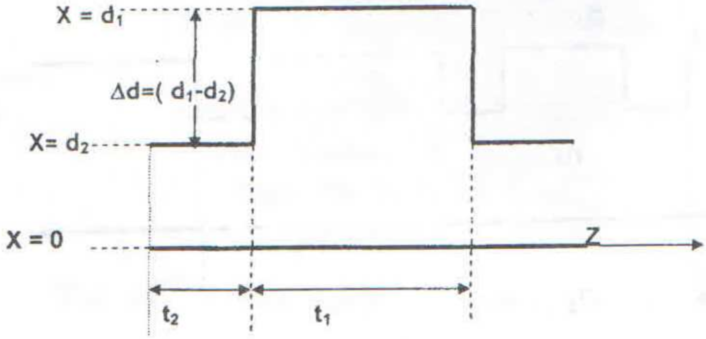


Figure 2 - A unit-cell of a corrugated DFB Waveguide .

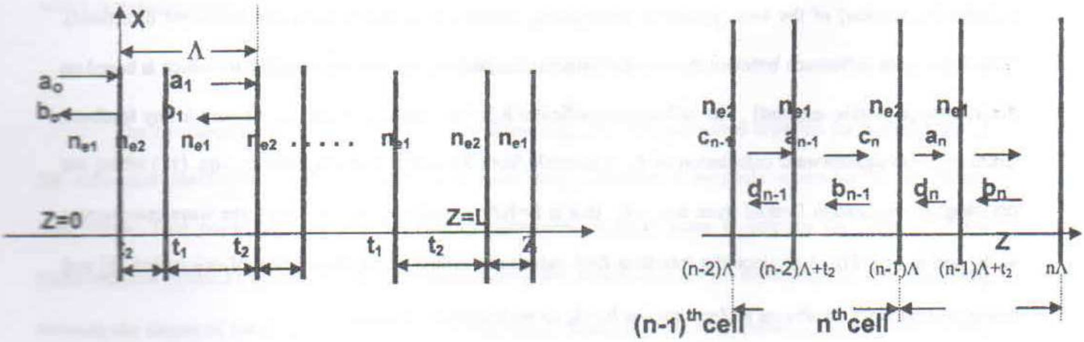
Where "j" is the imaginary unity and $\mathcal{E}_1(x)$ and $\mathcal{E}_2(x)$ are the transverse distributions of the mode fields in the respective sections of the unit-cell. The time-dependence $\exp(-j\omega t)$ will be dropped in what follows. The propagation constants are related to the free space wavelength λ by : $\beta_1 = k_o n_{e1} = (2\pi/\lambda) n_{e1}$ where k_o is the free-space wavenumber. The mode effective index n_{e1} in the thick section is the solution of the following eigenvalue equation [17]:

$$k_o d_1 (n_f^2 - n_{e1}^2)^{1/2} - \tan^{-1}\{(n_{e1}^2 - n_s^2)/(n_f^2 - n_{e1}^2)\}^{1/2} - \tan^{-1}\{(n_{e1}^2 - n_c^2)/(n_f^2 - n_{e1}^2)\}^{1/2} = m\pi \qquad (2\text{-a})$$

While the mode effective index $n_{e2} = \beta_2 / k_o$ is the solution of the following eigenvalue equation:

$$k_o d_2 (n_f^2 - n_{e2}^2)^{1/2} - \tan^{-1}\{(n_{e2}^2 - n_s^2)/(n_f^2 - n_{e2}^2)\}^{1/2} - \tan^{-1}\{(n_{e2}^2 - n_c^2)/(n_f^2 - n_{e2}^2)\}^{1/2} = m\pi \qquad (2\text{-b})$$

The integer " m" is the order of the mode (for the fundamental mode m=0). The two Eqns. (1-a) and (1-b) reflect the well known fact [16,18 and 19] that a mode can be viewed as an inhomogeneous plane wave propagating in a uniform homogeneous medium with refractive index equals to the mode effective index. And hence, the alternating thick and thin sections of the corrugated zone of the waveguide is regarded as a stack of multilayer dielectric media [16,5 and 6] with alternating refractive indices n_{e1}, n_{e2}, ...and so on, as shown in Fig. 3-a. If N is the total number of periods, then $L=N\Lambda$.



(a) (b)

Figure 3 - (a) A stack of dielectric layers equivalent to the corrugated section of a DFB waveguide.
(b) Forward and backward waves in two consecutive cells in the dielectric stack.

Within any cell (say the n^{th}-cell) there will be a forward wave (propagating towards positive Z) and backward wave (propagating towards negative Z), the Z-dependence of these waves is not of the form $\exp(\pm j\beta z)$ as in Eqns. (1-a) and (1-b); but will be [4-5] $a_n \exp(j\beta_1 z)$ for the forward wave within the thick section (thickness t_1 and denoted by t_1-layer) of the n^{th}-cell and $b_n \exp(-j\beta_1 z)$ for the backward wave. Within the thin section (thickness t_2 and denoted by t_2-layer) of the same n^{th}-cell, the forward and backward waves have a Z-dependence similar to the previous ones, namely: $c_n \exp(j\beta_2 z)$ and $d_n \exp(-j\beta_2 z)$ for the forward and backward where the elements T_{ij} of the translation matrix are as follows [5]:

$$T_{11} = \exp(-j\beta_1 t_1)\left\{ \cos \beta_2 t_2 - \{ (j \sin\beta_2 t_2 /2) [(\beta_2/ \beta_1) + (\beta_1/ \beta_2)]\} \right\} \quad (12)$$

$$T_{12} = \exp(-j\beta_1 t_1) \quad \{ (-j \sin\beta_2 t_2 /2) [(\beta_2/ \beta_1) - (\beta_1/ \beta_2)] \} \quad (13)$$

$$T_{21} = -T_{12} = \exp(-j\beta_1 t_1) \{ (j \sin\beta_2 t_2 /2) [(\beta_2/ \beta_1) - (\beta_1/ \beta_2)] \} \quad (14)$$

$$T_{22} = \exp(j\beta_1 t_1)\left\{ \cos \beta_2 t_2 + \{ (j \sin\beta_2 t_2 /2) [(\beta_2/ \beta_1) + (\beta_1/ \beta_2)] \right\} \quad (15)$$

It is worthy to note that in the coupled-modes formalism the reflections at the discrete interfaces between the dielectric layers are assumed small, and this allows the replacement of the difference Eqns. (5) to (8) (which are in fact the boundary conditions) by two coupled differential equations relating the rate of change (in the Z-direction) of the two oppositely propagating modes (i.e. in the forward and backward directions). This is the main difference between the two formalisms (coupled-modes and Floquet's theory which is based on the translation matrix method) . The reflection coefficient R_u of one unit cell is the key element in any feedback structure. A straightforward calculation of R_u is possible from the matrix relation given in Eqn. (11) when we consider a corrugation formed from one cell, that is $L=N\Lambda=\Lambda$ and $N=1$, so we have three wave components a_o, b_o and a_1 (c.f. Fig. 3-a) since the interface $Z=\Lambda$ recedes to infinity for a corrugation of one period [5] and hence the boundary conditions at $Z=\infty$ implies $b_1= 0$, so we immediately obtain:

$$R_u = (b_o/a_o)\Big|_{b_1=0} = (T_{21}/ T_{11}) = \frac{(j \sin\beta_2 t_2 /2) [(\beta_2/ \beta_1) - (\beta_1/ \beta_2)]}{\cos \beta_2 t_2 - \{(j \sin\beta_2 t_2 /2)[(\beta_2/ \beta_1) + (\beta_1/ \beta_2)]\}} \quad (16)$$

where a_o , b_o and b_1 are shown in Fig. 3-a. If the optical thickness of the t_2-layer is equal to a quarter wavelength, i.e. it satisfies the Bragg condition, then the reflection coefficient of that layer is maximum when :

$$\beta_2 t_2 = \pi/2 \quad (17\text{-a})$$

Similarly for the t_1-layer:

$$\beta_1 t_1 = \pi/2 \quad (17\text{-b})$$

Of course the matrix that relates c_{n-1} and d_{n-1} to c_n and d_n is different from that one given by Eqn. (11), however it is similar in form [5] . For a dielectric multilayer when the conditions for maximum reflection coefficient (17-a) and (17-b) are satisfied (i.e. $\sin\beta_2 t_2 = 1$ and $\cos \beta_2 t_2 = 0$) the reflection coefficient R_u of a unit-cell, according to (16), will be given by :

$$R_u = \frac{[(\beta_1/\beta_2) - (\beta_2/\beta_1)]}{[(\beta_1/\beta_2) + (\beta_2/\beta_1)]} = \frac{\beta_1^2 - \beta_2^2}{\beta_1^2 + \beta_2^2} = \frac{n_{e1}^2 - n_{e2}^2}{n_{e1}^2 + n_{e2}^2} \qquad (18)$$

Considering the fact that the mode effective indices n_{e1} and n_{e2} depend on the wavelength λ via the eigenvalue Eqns. (2-a) and (2-b), the maximum value of R_u Eqn. (18) can be optimized when:

$$(dR_u/d\lambda) = \frac{[n_{e2}^2 n_{e1} (dn_{e1}/d\lambda)] - [n_{e1}^2 n_{e2} (dn_{e2}/d\lambda)]}{[n_{e1}^2 + n_{e2}^2]^2} = 0 \qquad (19)$$

which is possible when :

$$[(dn_{e1}/d\lambda) / (dn_{e2}/d\lambda)] = n_{e1}/n_{e2} \qquad (20)$$

The condition given by Eqn. (20) is, to our knowledge, the first published criterion for optimization of the reflection coefficient of the unit-cell of a distributed feedback waveguide operating at the Bragg wavelength. That condition is in fact a two-fold eigenvalue criterion since it requires the solution of the transcendental equations (2-a) and (2-b) to find n_{e1} and n_{e2} and then search for the wavelength at which the ratio between the slopes of the dispersion curves $n_{e1}(\lambda)$ and $n_{e2}(\lambda)$ is equal to the ratio between the effective indices themselves. The optimization procedure will be outlined and checked numerically in the following section.

III. Verification and Discussion of the Optimization Condition:

Evidently, there is no closed-form analytic solution for (20) since this requires the solution of transcendental equations as stated previously. Only a numerical solution is possible. However, we can discuss the overall character of the solution and the optimization process by intuitive reasoning (which will be verified later on) : the guided mode suffers from reflections when it encounters the steps forming the corrugations (both the step-up and the step-down discontinuities in the corrugated zone of the waveguide). A rough estimation of the reflection coefficient at a step-like discontinuity is the plane-wave reflection coefficient

$(\beta_1 - \beta_2)/(\beta_1 + \beta_2)$. Furthermore, if the difference in the refractive indices between the film and the substrate is small, i.e. $(n_f - n_s)/n_f$ is much less than 1, then the reflection coefficient which can be written as $(\beta_1 - \beta_2 / 2\beta)$ increases as the difference $(\beta_1 - \beta_2)$ increases (where β is the average value of the propagation constants β_1 and β_2). This means that when the two small sections of the waveguide forming the unit-cell have quite different characteristics (thickness), that is one mode (for example in the thick section d_1) is far from cutoff while the other one (in the thin section d_2) is near the cutoff at the wavelength of interest λ_o . Accordingly, the optimization process can be done as follows: fix the values of n_c, n_f, n_s and λ_o , then choose d_1 (in principle arbitrary).One may choose it to satisfy certain requirement, for example for single-mode operation at the wavelength λ_o , the cutoff wavelength of the first-order mode, i.e. m=1 in (2-a), must be less than the wavelength of λ_o (say 60% λ_o) to ensure the single-mode operation (and far from cutoff) of the thick waveguide, i.e. :

$$d_1 = \frac{[\pi + \tan^{-1}\{(n_s^2 - n_c^2)/(n_f^2 - n_s^2)\}^{1/2}]}{(2\pi/0.6\lambda_o)\,[n_f^2 - n_s^2)\}^{1/2}]} \qquad (21)$$

Then, search for the thickness d_2 that satisfies the optimization condition (20) at the wavelength of interest λ_o by varying the ratio $q=d_2/d_1$ (i.e. varying the thickness d_2) at small steps Δq and for each value of "q" we solve the eigenvalue Eqns. (2-a) and (2-b) numerically in a wavelength range around λ_o . Then calculate numerically the slopes of the dispersion curves $S_1=d[n_{e1}(\lambda)]/d\lambda$ and $S_2= d[n_{e2}(\lambda)]/d\lambda$ and check for the optimization condition (20) until it is satisfied at that wavelength λ_o. Once the ratio "q" that meets the requirements in (20) is found, the widths t_1 and t_2 of the thick and thin sections of the corrugations can be calculated according to Bragg conditions (17-a) and (17-b). To check numerically the existence of an optimum value for R_u, we considered a waveguide having $n_c=1$, $n_f=3.61$, $n_s=3.6$, the wavelength of interest $\lambda_o = 1.5\mu m$ and $d_1= 2.47\mu m$ (according to (21)).

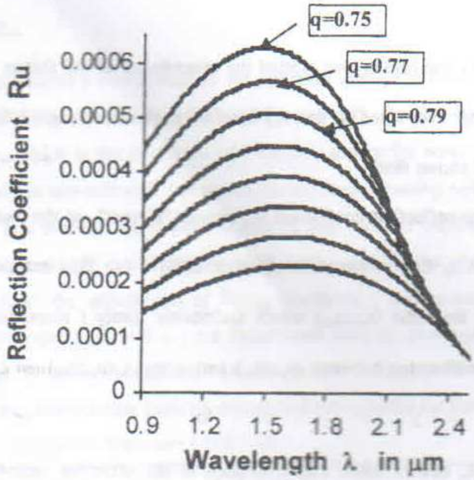


Figure 4 - Variations of the unit cell reflection coefficient R_u against the variations of the wavelength λ for different values of the parameter $q=(d_2/d_1)$ around the wavelength of interest $\lambda_o = 1.5\mu m$.

Fig. 4 shows that R_u has a peak value when q=0.75 which corresponds to d_2= 1.85μm. The location of the peak of R_u shifts towards the longer wavelengths as "q" increases beyond the value 0.75; and the magnitude of that peak decreases as "q" increases since the amount of reflections decreases as the dimensions of the two sections forming the unit-cell become close to each other (i.e. "q" approaches unity when d_2 approaches d_1). To verify the condition (20), we solve numerically (for q=0.75) the eigenvalue Eqns. (2-a) and (2-b), to find n_{e1} and n_{e2}, and calculate the slopes S_1 and S_2 of the dispersion curves $n_{e1}(\lambda)$ and $n_{e2}(\lambda)$. The reflection coefficient R_u and the difference in effective indices $\Delta n_e = n_{e1}(\lambda) - n_{e2}(\lambda)$ are also calculated and plotted in figure 5.

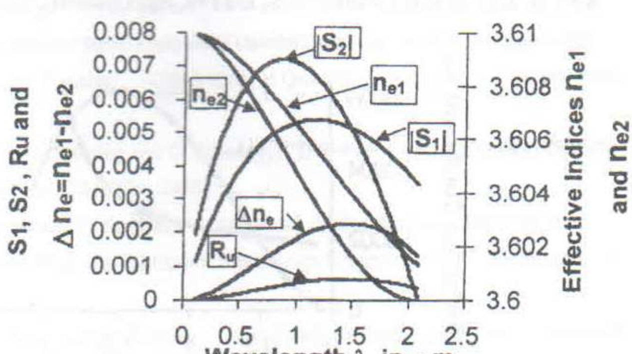


Figure 5 - Dispersion curves $n_{e1}(\lambda)$ and $n_{e2}(\lambda)$ of the fundamental modes in the two sections of a unit-cell. The slopes S_1 and S_2 of the dispersion curves intersect at the desired wavelength λ_o=1.5μm. The point of optimum R_u.

To discuss conveniently the results, we plotted the magnitudes of the slopes instead of the slopes S_1 and S_2 themselves (which are negatives since n_{e1} and n_{e2} decrease as the wavelength λ increases).

A careful investigation of Fig. 5 shows that:

1- The optimum value of the reflection coefficient R_u occurs "almost" at the point of intersection of the slopes S_1 and S_2, i.e. when $S_1=S_2$ (or alternatively when $S_1/S_2 = 1$) . This is expected, because according to (20), S_1/S_2 is equal to the ratio (n_{e1}/n_{e2}) which is "nearly" unity (since $\Delta n_e = n_{e1} - n_{e2} \approx 0.002$ at λ_o=1.5μm and hence the difference between (n_{e1}/n_{e2}) and unity is $(n_{e1}/n_{e2})-1= (\Delta n_e/n_{e2}) < 0.002$, that is $(n_{e1}/n_{e2}) \approx 1$.

2- The optimum value of R_u occurs when the difference in the effective indices Δn_e is maximum (as discussed "intuitively" at the beginning of this section). This is confirmed since the peaks of Δn_e and R_u occur at the same wavelength (1.5μm). To get a wider look on the behavior of the above mentioned quantities, we performed the calculations in a wavelength range beyond the single-mode limit of the thick waveguide,i.e. shorter than 0.6 λ_o = 0.9μm, while the long wavelength limit is determined by the cutoff wavelength of the fundamental mode λ_{co} (i.e. m=0 and $n_e= n_s$, in eq.(2-a)) of the thick waveguide section of the corrugations, which is given by :

$$\lambda_{co} = 2\pi d_1 [n_f^2 - n_s^2]^{1/2} / [\tan^{-1}\{(n^2_s - n^2_c)/(n^2_f - n^2_s)\}^{1/2}] \quad (22)$$

To emphasize the condition (20) we plotted separately in Fig. 6 the reflection coefficient R_u and the ratio S/F (i.e. equ.(20) itself) where $S=S_1/S_2$ and $F=n_{e1}/n_{e2}$. Obviously, R_u attains its peak value when S/F=1.

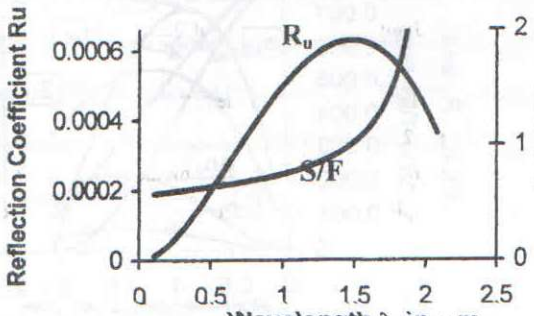


Fig. 6- Variations of the unit-cell reflection coefficient R_u against the variations of λ. The peak of R_u occurs when S/F = 1, in agreement with the optimization condition (20).

IV. Conclusions :

In this paper we introduced a new condition for optimization of the reflection coefficient of DFB waveguides. It is shown that the optimization of the reflection coefficient of a unit-cell of a distributed feedback waveguide is possible. The translation matrix formalism, which is the backbone of Floquet's theory for wave propagation in periodic media, is used to relate the electromagnetic field in one cell with that one in the next neighbouring cell.

The optimization condition is a two-fold eigenvalue equation, and hence it implies a numerical solution. We discussed the general character of the solution and explained the optimization procedure which has been verified numerically. That condition must be met "before" the satisfaction of Bragg condition (quarter-wave thickness) since the lengths of the sections of the corrugated waveguide (t_1 and t_2) are determined once the propagation constants are known. The method is accurate and can be used for the optimization, analysis, design and assessment of many distributed feedback-based devices and components for optical communications such as: corrugated waveguides for laser diodes, optical amplifiers, narrow-band filters and dense wavelength division multiplexers (DWDM).

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